Rømer Revisited

A Modern Estimation of the Speed of Light from Observations of Jupiter’s Galilean Satellites

# Abstract

Ole Rømer (1644-1710) is remembered nowadays for establishing, from observations of eclipses of the Galilean satellites of Jupiter, that the velocity of light is finite. The paper describes the use of amateur timings of Galilean eclipses observed during 2012-14 to recreate Rømer’s work together with a theoretical investigation of the accuracy of such approaches to estimating the speed of light.

# Introduction

Ole Rømer (1644-1710) is one of the key figures of astronomy, remembered nowadays for establishing that the velocity of light is finite. He based the conclusion on nine years (1668-76) of observations of the Galilean satellites of Jupiter, made at Uraniborg and Paris, and communicated it to the Académie Royale des Sciences in Paris in 1676.



Figure 1. Ole Rømer by Jacob Coning, c. 1700.

The four Galilean satellites (Io, Europa, Ganymede and Callisto, in order outwards from Jupiter) present a fascinating spectacle. Their orbits are approximately coplanar with that of Jupiter and, as a result, they can transit in front of the planetary disk, are subject to occultations when they pass behind the planet, and may enter its shadow and be eclipsed. For Io and Europa, prior to opposition, eclipse disappearance events are visible but corresponding reappearance events occur when the satellite is occulted and hence are invisible; after opposition, the situation is reversed, eclipse disappearance events occur in occultation and are invisible, but corresponding reappearance events are visible. Ganymede, orbiting further from the planet, follows essentially the same pattern but occasionally both disappearance and reappearance phenomenon of an eclipse are visible. Callisto, orbiting even further from Jupiter, exhibits behaviour similar to Ganymede and, in addition, throughout lengthy periods during each apparition of the planet, passes consistently above or below the shadow cone, missing eclipse altogether.

Rømer’s approach was based on an assumption that eclipse disappearances and reappearances of Io, observed by a hypothetical Jovicentric [1] observer, follow a regular periodicity. From this, he developed an argument to conclude that light does not travel instantaneously; there is, however, no record that he estimated its velocity. It is a comparatively simple matter to repeat his approach and, using modern information about the scale of the solar system, to take a step further and estimate the speed of light (denoted *c*) but, surprisingly, few astronomers have reported tackling the challenge. However, in early 2012, members of the Orwell Astronomical Society, Ipswich (OASI), began the project. Independently, in late 2013, members of the Hampshire Astronomical Group (HAG) began a similar project and, in 2014, the organisers of National Astronomy Week 2014 (NAW 2014) encouraged widespread participation and reporting of results to add to the observations obtained by members of OASI and HAG.

This paper describes the observations and their analysis to estimate *c*. (In fact, because of the method adopted, it is often more natural to estimate the quantity 1/*c*, expressed as minutes of light time for unit distance 1 AU.) An analysis based on modern, accurate predictions of the positions of the Galileans yields a remarkably accurate result, but provides little insight into the orbital mechanisms involved. Conversely, an analysis following the approach of Rømer produces a range of estimates of *c* of varying accuracy, and provides insight into the main orbital effects at play, when the method can be expected to work, and when it is likely to fail. Finally, observations by Rømer and the French astronomer Jean Picard (1620-82) are analysed. Although individually they do not agree well with the predictions of modern ephemerides, collectively they can be analysed to produce an accurate estimate of *c*; the paper closes with an estimate that Rømer himself could have produced, had the scale of the solar system been understood in his era.

# Historical context

In 1671, Picard travelled to the observatory of the late Tycho Brahe (1546-1601) at Uraniborg to copy Brahe’s observing reports and determine the exact location of the site. Rømer, who had been working with Brahe’s manuscripts, was detailed to provide assistance. He applied himself so successfully to the task that, when Picard returned to France, he arranged for Rømer to accompany him and take up employment at the Observatoire de Paris under the directorship of Giovanni Domenico Cassini (1625-1712).

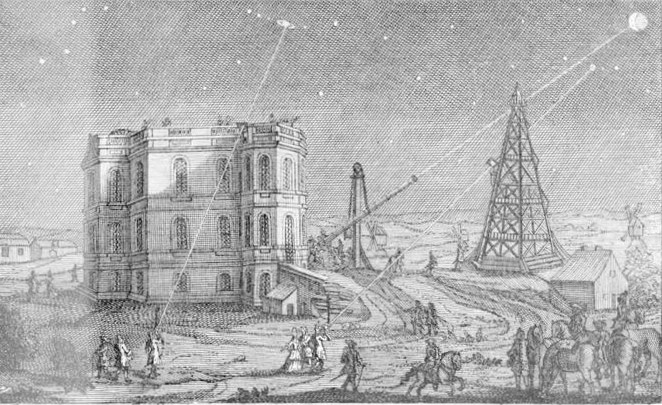


Figure 2. Observatoire de Paris in the 17th Century.

Cassini had begun his astronomical career in Bologna where, among other topics, he studied the Galilean satellites, publishing the first reasonably accurate tables of their motions, *Ephemerides Bononienses Mediceorum Siderum* [2]. The following year, he moved to work at the Observatoire de Paris under the auspices of the Académie Royale des Sciences, continuing his study of Jupiter and its satellites. Cassini’s tables enabled Rømer to formulate his conclusion.

Rømer announced his discovery to the Académie Royale des Sciences on 1676 November 21. Unfortunately, he did not publish the means by which he arrived at his finding and, although it was reported in the *Journal des Sçavans* [3] in December of the same year (and in translation in the *Philosophical Transactions* [4] the following year), the report lacks detail and is somewhat unclear, limiting understanding of the details of the work.

However, the broad thrust of Rømer’s approach is clear: it was based on geometric considerations illustrated in figure 3 (redrawn from the *Journal des Sçavans*). A represents the Sun and B Jupiter. The circle EFGHLK represents the Earth’s orbit. The circle around Jupiter represents the orbit of Io; the satellite enters Jupiter’s shadow and is eclipsed at C and leaves the shadow and reappears at D. Eclipse disappearance times and reappearance times are assumed to be periodic. The point L is close to second quadrature, the moment after opposition when, seen from the Earth, the Sun and Jupiter are at 90°, at which time the distance between the planets is increasing at its greatest rate. Suppose that with the Earth at L, Io is at D, just emerging from the shadow of Jupiter. After some 42.5 hours, Io completes a revolution around Jupiter and the Earth has moved to point K. If light does not travel instantaneously, the second emergence of Io at D will appear late, as seen from Earth, due to the increased distance from Jupiter. Around the time of first quadrature, on approach to opposition, at points F and G, the situation is reversed and immersions of Io into the shadow of Jupiter at point C appear early. The effect might not be apparent over a single revolution of Io but Rømer believed that it would be cumulative, and certainly visible over a period of 40 or more revolutions of the satellite.

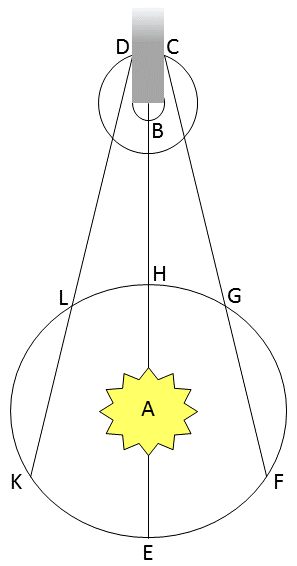


Figure 3. Diagram explaining Rømer’s work.

The conclusion did not meet immediate universal acceptance. In the early 1670s, Cassini himself had proposed, based on observations of the Galilean satellites, that the speed of light was finite; however he later rejected the idea and thus remained hostile towards Rømer’s conclusion. Kepler, Hooke and Descartes also believed that light travelled instantaneously. However, many prominent figures such as Newton, Halley, Flamsteed and Huygens [5] quickly accepted Rømer’s findings. Over time, evidence amassed to support his conclusion until, in 1729, James Bradley (1693-1762), third Astronomer Royal, announced the discovery of the aberration of light, and the matter was well and truly settled.

# Modern observations

It is a simple matter to follow Rømer’s approach and, using modern information concerning the scale of the solar system, to estimate the speed of light. A modest telescope will show the Galileans entering and leaving eclipse and, because the scale of the orbits of the Earth and Jupiter accounts for differences in light travel time between the two of up to approximately 21 minutes, crude methods of timing (e.g. eye and stopwatch) are adequate to reveal the variability. Indeed, former president of the BAA, Dr Robert Atkinson, in an address to an ordinary meeting of the Association in 1961, speaking of Rømer’s method, stated [6]: *Incidentally, this too is an observation which you can yourselves repeat; the method cannot give any approach to a precise figure for the speed of light, but you will find it instructive…* Despite the accessibility of the project and its historical resonance, it seems that few amateur astronomers have followed Rømer: the *JBAA* contains no reports of attempts to repeat his method and a search of the SAO/NASA Astrophysics Data System in 2014 May identified only one such report [7]. Unfortunately, too, the efforts of the organisers of NAW 2014 to encourage participation turned out to be less effective than had been hoped: by the close of NAW 2014, only members of OASI and HAG, who had independently begun the project, and one of the organisers of NAW 2014 himself, had submitted observing reports.

Table 1 lists the observations. Although Rømer’s method relies on timings of eclipse phenomena of Io, several observers reported timings of eclipses of other satellites and of transits and occultations. All timings were obtained by eye and stopwatch techniques apart from two obtained by offline analysis of video recordings. (Note that some of the observations by members of HAG were obtained by multiple co-operating observers reporting a single event time: in such cases, only the lead observer is listed below. Participating members of HAG not listed below were Tim Nelson, Carol Bryan, Peta Bosley and Steve Rogers.)

Because the Galilean satellites are of appreciable size (their radii range from 1815 km for Io to 2631 km for Ganymede [8]) eclipse disappearances and reappearances are far from instantaneous. For disappearances, the time reported is the last instant at which the Galilean was visible to the observer; for reappearances, the first instant at which it was visible. No attempt was made to estimate the time of mid-phenomenon.

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| --- | --- | --- |
| **Observer & Affiliation** | **Telescope(s)** | **Observations** |
| James Appleton  OASI | 260mm refractor,  250mm Schmidt-Cassegrain,  200mm reflector. | Io: 1 D 2012 November 1, 1 R 2012 December 10.  Europa: 2 R 2012 December 8, 2013 January 16.  Ganymede: 3 D 2012 January 15 – 2012 November 3, 3 R 2012 February 19 – 2012 December 9. |
| Steve Bosley  HAG | 60mm & 178mm refractors, 200mm & 610mm reflectors. | Io: 6 D 2013 November 29 – 2013 December 22, 6 R 2014 January 23 – 2014 May 18, 3 occ D 2014 February 13 – 2014 March 8, 1 occ R 2013 November 22, 1 SoT 2014 March 9, 1 EoT 2014 January 22. |
| Steve Broadbent  HAG | 610mm reflector | Io: 1 D 2013 November 29. |
| Martin Cook  OASI | 250mm Newtonian | Io: 3 D 2012 November 1 – 2013 December 22, 8 R: 2013 January 4 – 2014 May 18.  Europa: 2 R 2013 January 9, 2013 January 16.  Ganymede: 2 D 2012 February 19, 2012 November 3. |
| Steve Futcher  HAG | 178mm refractor, 200mm reflector. | Io: 3 D 2013 November 29 – 2013 December 22, 1 R 2014 May 18, 1 occ D 2014 February 22, 1 occ R 2013 December 8. |
| Andy Gibbs  OASI | 200mm ACF reflector | Io: 2 R 2014 January 23, 2014 March 17. |
| Roy Gooding  OASI | 140 mm Maksutov-Cassegrain | Io: 1 D 2012 November 1. |
| Mike O’Mahoney  OASI | 71mm & 120mm refractors, 300mm reflector. | Io: 5 R 2013 January 27 – 2014 March 24.  Europa: 1 R 2013 March 21, 1 occ R 2013 December 9.  Ganymede: 1 R 2014 March 27. |
| Neil Morley  OASI | 100mm refractor, 200mm & 250mm reflectors. | Io: 2 R 2014 February 22, 2014 March 1, 2 occ D 2014 March 8, 2014 March 24.  Europa: 1 R 2014 March 15, 1 occ D: 2014 March 29.  Callisto: 1 occ R 2014 March 19. |
| Gerry Pilling  OASI | 125mm Schmidt-Cassegrain, 200mm Newtonian. | Io: 4 R 2014 March 1 – 2014 May 2, 2 occ D 2014 February 22, 2014 March 24.  Europa: 1 R 2014 January 17.  Callisto: 1 D 2013 November 22. |
| Robin Scagell  NAW 2014 | 80mm refractor | Io: 1 R 2013 January 10 (v). |

Table continued.

|  |  |  |
| --- | --- | --- |
| **Observer & Affiliation** | **Telescope(s)** | **Observations** |
| Alan Smith  OASI | 250mm Newtonian | Io: 4 D 2012 November 1 – 2013 December 29, 10 R 2012 December 3 – 2014 March 24.  Europa: 3 R 2013 January 9 – 2013 March 21.  Ganymede: 3 D 2012 February 19 – 2013 March 12, 2 R 2012 December 16, 2013 March 5. |
| Joe Startin  OASI | 150mm reflector | Io: 2 D 2012 November 1, 2013 December 29, 6 R 2012 December 10 – 2014 March 24, 1 occ D 2013 March 5.  Europa: 1 R 2013 January 16.  Ganymede: 2 R 2012 November 3, 2013 March 5. |
| Chris Stevens  OASI | 130mm reflector | Io: 1 D 2013 December 22. |
| Paul Whiting, FRAS  OASI | 250mm Newtonian | Europa: 1 occ R 2013 December 9. |
| Mike Whybray  OASI | 114mm reflector | Io: 1 D 2013 November 29 (v), 2 R 2014 March 1, 2014 May 18.  Europa: 1 R 2014 March 22, 1 occ R 2013 December 9. |

**Table 1. Observations.**

D = eclipse disappearance (immersion), R = eclipse reappearance (emersion), occ D = occultation disappearance, occ R = occultation reappearance, SoT = start of transit, EoT = end of transit, v = timing by offline analysis of video recording.

# Analysis

The analysis was restricted to timings of eclipses, of which there were 100 observations. Fourteen were compromised by cloud or other factors causing uncertainty; excluding them from further consideration, the analysis proceeded upon the remaining 86, summarised in table 2.

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| **Satellite** | **Observations** | **Dates** |
| Io | 19 D  42 R | 2012 November 1 – 2013 December 29  2012 December 3 – 2014 May 18 |
| Europa | 9 R | 2013 January 9 – 2014 March 22 |
| Ganymede | 8 D  8 R | 2012 January 15 – 2013 March 12  2012 February 19 – 2014 March 27 |

**Table 2. Eclipse timings analysed.**

D = eclipse disappearance, R = eclipse reappearance.

A direct approach to estimating the speed of light from the observations proceeds as follows. Let CJ be the calculated (predicted) time of a Galilean eclipse phenomenon (disappearance or reappearance) seen by a hypothetical Jovicentric observer. Let OE be the corresponding observed time on Earth. Let e be the apparent geocentric distance of Jupiter at the time of observation. Then simply plot the difference d=OE-CJ versus e, i.e. the delay experienced by a terrestrial observer in observing the phenomenon versus the distance to the planet. In practice, times of eclipse phenomena calculated for geocentric observers, CE, are widely available but the same is not true for Jovicentric observers and it is therefore necessary to calculate CJ as the predicted time of the phenomenon seen from the Earth minus the light time to Jupiter, T, i.e. CJ=CE-T.

Figure 4 shows the results of this approach, plotting d versus e. Times of mid-phenomenon listed in BAA Handbooks [9, 10, 11] were used as values of CE. Geocentric distances of Jupiter and light times to the planet were calculated using the NASA JPL ephemeris DE-405. Linear regression lines were fitted to the data, with equations as shown. The gradient terms of the regression lines provide estimates of the light time for unit distance, 1/*c*; inverting and changing units provides estimates of *c*.

**Figure 4. Observed minus calculated eclipse times versus Earth-Jupiter distance.**

Several approximations and assumptions are involved in this approach: the effect of light times up to a few seconds across the Jovian system is ignored; for each Galilean, it is assumed that observers are consistent in reporting the first instant of reappearances, and are also consistent in reporting the final instant of disappearances; and of course it is assumed that the speed of light is known in so far as is necessary to formulate estimates of Jovicentric event times.

Predictions [12] in BAA Handbooks refer to times of mid-phenomenon. Generally, observers discerned the last glimpse of a satellite entering eclipse after mid-time, and the first glimpse of a satellite emerging from eclipse before mid-time. This results in an offset between the regression lines for disappearance and reappearance events for each satellite, represented by the constant terms. The lower orbital angular velocity of Ganymede means that it takes longer than Io to pass through the edge of Jupiter’s shadow, resulting in its disappearance and reappearance lines being spaced further apart. Table 3 summarises the results and includes a compound estimate obtained by weighting, according to number of observations, and adding the individual estimates.

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| --- | --- | --- | --- |
| **Phenomenon** | **No. Obs.** | **1/*c* (min/AU)** | ***c* (km/s)** |
| Io D | 19 | 7.69 | 324,000 |
| Io R | 42 | 8.58 | 291,000 |
| Europa R | 9 | 8.75 | 285,000 |
| Ganymede D | 8 | 8.72 | 286,000 |
| Ganymede R | 8 | 7.26 | 343,000 |
| **Compound** | **86** | **8.29** | **301,000** |

**Table 3. Estimates of light time for unit distance and speed of light.**

The accepted values of light time for unit distance and the speed of light are respectively 8.3167 min and 299,792 km/s. Given the diversity of telescopes used, the unsophisticated nature of the timing technology and the wide range of skills and abilities of the observers, the individual estimates, and especially the compound final estimate, are surprisingly accurate.

An analysis searching for a correlation between telescope aperture and reported time of emersion/ immersion, contrary to expectations did not provide a positive response. This was in part because of the limited number of observers and observations. Further, most observers tended to use a very small range of instruments, often just one, throughout the project, so that the effects of aperture were confounded with the skill and experience of the observer.

# Analysis following Rømer

The above analysis provides little insight into the various factors at play and it is instructive to eschew modern knowledge of the orbital dynamics of the Galileans and instead examine the observations using an approach modelled closely on that of Rømer, assuming that eclipse disappearances and reappearances occur periodically for each satellite. Thus, consider separately the five datasets of observations for Io disappearances, Io reappearances, Europa reappearances, Ganymede disappearances and Ganymede reappearances. For each dataset, proceed as follows:

1. Baseline the analysis relative to the earliest observation of the dataset.
2. Calculate the expected time, C, of each eclipse event assuming that it occurs after the earliest by an integer multiple of the synodic period, P, of the Galilean. (Calculate P from the sidereal period in [13].)
3. For each observed event time, O, calculate the difference, d = O – C. d represents the delay experienced by a terrestrial observer in witnessing the event, relative to the delay associated with the first event of the data set.
4. For each apparition of the planet, plot d versus the distance, e, between Jupiter and the Earth (calculated using the ephemeris DE-405).
5. Perform a simple linear regression to fit a straight line to the plot for each apparition.
6. As previously, the gradient of the regression line provides an estimate of 1/*c*.

The above algorithm, termed “Method A”, is a modern interpretation of how Rømer could have estimated the speed of light, based on the description in the *Journal des Sçavans*, had knowledge of the scale of the solar system been available in his era. Figure 5 shows the results (solid lines are used to indicate the temporal order of observations and regression lines are dotted) and table 4 provides a summary of estimates of 1/*c*.

**Figure 5. Plots of d versus e for the observations.**

The observations of Io comprise disappearance and reappearance events during two apparitions of Jupiter. Disappearance data consists of a cluster of five observations on 2012 November 1 during the earlier apparition, and a month’s worth of observations (2013 November 29 – 2013 December 29) during the later. During the earlier apparition, the presence of data for only a single event means that Method A cannot be applied. During the later apparition, the limited timespan of data and its variability compromise the method and the gradient of the regression line is wide of the mark. The analysis is much more satisfactory for reappearance observations of Io, for which there are 18 observations (2012 December 3 – 2013 May 6) during the earlier apparition and 24 (2014 January 23 – 2014 May 18) during the later: both regression lines fit the data well and the gradients are good approximations to the accepted value of 1/*c*. Observations of Europa also span two apparitions of Jupiter, and comprise only reappearance events. Data for the earlier apparition comprises seven observations of three events (2013 January 09 – 2013 March 21); the fit of the regression line is good but its gradient is a very poor approximation to 1/*c*. For the later apparition there are only two data points (2014 March 15 and 2014 March 22), so the question of goodness of fit of the regression line does not arise, and again its gradient is a poor estimate of 1/*c*. Observations of Ganymede comprise disappearances and reappearances during three apparitions of the planet. For disappearances, there are eight data points spanning four eclipse events across two apparitions, the regression lines fit the data well, one estimate of 1/*c* is reasonable but the other is poor. For reappearances, there are also eight observations, but spread across three apparitions of Jupiter; however, the earliest and latest apparition each contain only one observation and, although these are plotted in figure 5, the analysis is restricted to the six observations in the middle apparition. The regression line fits the data well and its slope provides a poor estimate of 1/*c*.

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| --- | --- | --- | --- | --- |
| **Satellite** | **D or R** | **Observations** | **1/*c* (min/AU)** | ***c* (km/sec)** |
| Io | R | 2012 December 3 – 2013 May 6  2014 January 23 – 2014 May 18 | 8.06  6.53 | 309,000  382,000 |
| Io | D | 2013 November 29 – 2013 December 29 | 4.45 | 560,000 |
| Europa | R | 2013 January 9 – 2013 March 21  2014 March 15 – 2014 March 22 | 27.8  22.6 | 90,000  110,000 |
| Ganymede | R | 2012 November 3 – 2013 March 5 | 26.1 | 95,000 |
| Ganymede | D | 2012 January 15 – 2012 February 19  2012 November 3 – 2013 March 12 | 26.8  11.7 | 95,000  213,000 |

**Table 4. Estimates of light time for unit distance and speed of light.**

The estimates depend critically on the assumed synodic periods of the Galileans. Table 5 lists accepted values of the synodic periods and estimates obtained from analysis of the observations by calculating, for each Galilean, separately for disappearance and reappearance events, the average time between consecutive observations divided by the average number of orbits. In the case of Io and Ganymede, estimates for disappearances and reappearances are weighted by the number of observations then combined. The estimates obtained from this simplistic approach differ from accepted values by only a few seconds for Io and Europa and by slightly more than a minute for Ganymede [14].

A more sophisticated analysis of the observations to estimate synodic periods, incorporating a correction for the light time (assumed known) between Jupiter and the Earth, effectively estimating the synodic period witnessed by a hypothetical Jovicentric observer rather than a terrestrial one, in fact produces estimates considerably further from accepted values. This may be because the observations are all atypical: Jupiter in its orbit was shortly after perihelion (2011 March 17) thus moving faster than average and, during the period, the inclination of the planet’s axis towards the Sun decreased steadily by 2°, tending to cause the length of eclipses to increase and times between successive reappearances, and between successive disappearances, to decrease. Correcting for the light time to the Earth removes a factor which may have partly masked these trends.

|  |  |  |
| --- | --- | --- |
| **Satellite** | **Accepted Value** | **Empirical Estimate** |
| Io | 1d 18h 28m 35.97s | 1d 18h 28m 38.36s |
| Europa | 3d 13h 17m 53.85s | 3d 13h 18m 04.70s |
| Ganymede | 7d 3h 59m 36.31s | 7d 4h 0m 44.0s |

**Table 5. Synodic periods.**

In any event, using the empirical estimates of table 5 in the analysis produces significantly different estimates of 1/*c*. (Differences compared to table 4 range from 20% to 240%.)

At this stage, it is apparent that Method A works *to an extent*. The available data shows that regression lines fit well plots of d versus e within an apparition of the planet. However, the intercept of the lines varies significantly between apparitions and, at least for Europa and Ganymede, the gradient can be a very poor estimate of 1/*c*. Clearly, further investigation is warranted, for which a larger and more robust set of observational data is required. Further exploration therefore considered the set of 548 high-precision timings reported by Mallama *et al* [15] of Galilean eclipses during the period 1990 November to 2009 December. The authors used a sophisticated approach to time eclipses, fitting a synthetic light curve to photometric data for each event, taking structural account of the effects of atmospheric refraction at the limb of Jupiter and topographically-induced satellite albedo variations; from the fitted curve they determined the time of mid-phenomenon. Figure 6 shows an analysis, following steps 1-4 of Method A, of two portions of the data, Europa reappearances between 1991 March 22 and 2002 May 16 and Ganymede disappearances between 1992 May 11 and 2001 September 12. (These periods are of no special significance and simply illustrate clearly some of the effects visible in the whole.) The lines on the graphs indicate the temporal order in which observations were made; they are not regression lines fitted to the data.

Consider first the figure for Europa. The plot starts at an apparent geocentric distance of 4.7 AU with d=0 (by construction) and the next point is at distance 4.5 AU with d=-17.3 minutes. d then grows increasingly large and negative, following no obvious pattern, until it attains a value of almost ­131 minutes. Thereafter, d exhibits segments of approximately linear increase, but with a gradient typically approximately twice that of the expected value, 8.3 min/AU. (d has the dimensions of time. The fact that it grows to such a large, negative value is of no special significance as the baseline defined by the first observation of each data set is arbitrary.)

**Figure 6. d vs e for Europa reappearances 1991 March 22 – 2002 May 16, and for Ganymede disappearances 1992 May 11 – 2001 September 12.**

For Ganymede, the data shows a much greater range and variability, and there are only four segments of three or more consecutive points where d varies approximately linearly (judged by eye), two increasing as a function of distance, two decreasing.

The portions of the graphs exhibiting an approximately linear relationship between d and e, where the gradient is close to the expected value, 8.3 min/AU, represent domains within which Method A can be said to *work*. Portions where there is no such relationship are clearly influenced by factors not accounted for in the simple formulation of the Method. In fact, across the entire data set of 548 observations, there is a clear trend: Io exhibits the most regular behaviour, with observations generally exhibiting an approximately linear increase in d with e, with a gradient close to 8.3 min/AU. In progression through Europa, Ganymede and Callisto, the regularity of results, the extent to which d varies linearly with e and, where the relationship is (approximately) linear, the closeness of the gradients to the expected value, all deteriorate progressively.

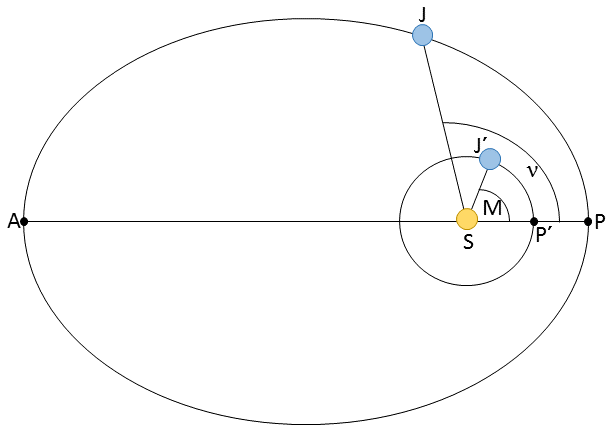
The analysis demonstrates that factors in addition to light travel time indeed significantly affect the apparent regularity of eclipses, and that their influence is variable and more pronounced for the outer satellites. The main such factors are: the ellipticity of Jupiter’s orbit, the inclination of the orbit of the satellite to that of the planet, and inherent variation in the orbital motion of the satellite (through mutual gravitational resonances and the effect of perturbations by other bodies in the solar system). It is possible to make an order of magnitude estimate of the effect of each as follows.

**Eccentricity of Jupiter’s orbit**

The eccentricity of Jupiter’s orbit, 0.0485, is large enough to cause a significant deviation in the motion of the planet from its mean rate. A crude estimate can be constructed as follows of the maximum amount by which this advances or retards eclipse phenomenon. The explanation is cast in terms of Io, but is applicable also to the other Galileans.

Refer to figure 7 (not to scale). Ellipse PJA represents the orbit of Jupiter, an ellipse with the planet at J. Line PP’SA is the major axis of the ellipse. Perihelion is at P; aphelion at A. The Sun is at S, one focus of the ellipse. The true anomaly, ν, is defined as ∠PSJ. Consider now a fictitious Jupiter, J’, describing a circular orbit around the Sun, with constant angular velocity, and the same period as the true Jupiter. Suppose that this body is at P’, on the line SP, when Jupiter is at P. The mean anomaly, M, is defined as ∠P’SJ’.

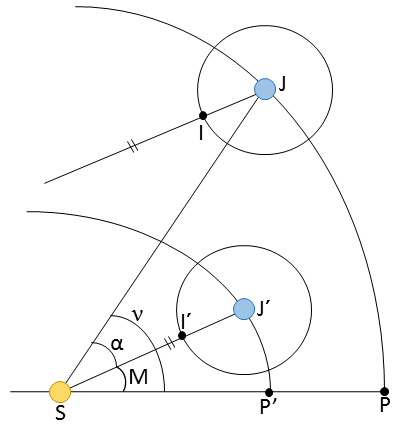
It is a standard exercise in orbital mechanics to compute ν from M, given knowledge of the eccentricity and period of the orbit – see for example [16].



**Figure 7. Orbit of Jupiter.**

Consider now figure 8, which builds on the above and adds detail of the orbit of Io around Jupiter to illustrate the effect of the planet’s orbital eccentricity on the time of eclipse phenomena. Let T be the synodic period of Io. Suppose that, when the real Jupiter is at P and the fictitious Jupiter is at P’, Io is in conjunction with the Sun as seen by a Jovicentric observer (or the moon is in any other defined configuration with respect to the Sun). Then at a time later by an integer multiple of T, the fictitious Jupiter has moved to J’, the mean anomaly is M, and Io, at position I’, is again in conjunction with the Sun. However, the real Jupiter has moved to J, the true anomaly is ν, and Io, at position I, has not returned to conjunction with the Sun. (As illustrated, near perihelion, the angular velocity of Jupiter in its orbit is greatest, the path PJ is longer than PJ’, and Io needs to revolve further around its orbit before coming into conjunction with the Sun again.)

Let α=ν-M, the difference between true and mean anomalies. Assuming that all bodies and orbits are coplanar, by virtue of alternate angles, α also represents the additional angle through which Io must revolve to return to conjunction with the Sun.



**Figure 8. Effect of eccentricity of Jupiter’s orbit.**

Tabulating values of M and ν throughout the range 0-360° shows that the maximum and minimum values of α are +5.6° and -5.6° respectively. Expressing the quantities as fractions of the sidereal period of Io, it follows that the maximum advancement and retardation due to the eccentricity of Jupiter is approximately 40 minutes.

**Inclination of satellite’s orbit**

The inclination of Jupiter’s equator to its orbit is 3.12° and the inclinations of the orbits of the Galileans to the equator of the planet range from the smallest, 0.04° for Io, to the largest, 0.51° for Callisto. The net effect is to make passages of the Galileans through the umbral shadow cone in general non-central and, the further a passage is from central, the later eclipse begins and the earlier it ends. Indeed, as noted previously, although the three inner Galileans enter eclipse every time they pass behind the planet, the outermost, Callisto, orbits at sufficient distance that it displays “eclipse seasons”, periods when it does consistently enter eclipse, alternating with periods when it does not, instead passing above or below the shadow cone, avoiding eclipse [17].

Some simplifications are helpful to estimate the maximum effect of the inclination of the Galilean orbit on eclipse times. Thus, assume that the planet is at its mean distance from the Sun (the solar distance affects the size of the umbral shadow cone), and that the satellite travels in a uniform, circular, unperturbed orbit in the equatorial plane. For Io, Europa and Ganymede, simple geometrical calculations then enable estimation of the time spent in the planet’s umbral shadow when the passage is central and when the passage is at its most northerly or southerly (north pole of Jupiter’s axis pointed respectively towards and away from the Sun). The resulting maximum perturbation to eclipse times is half of the difference between the two estimates. For Callisto, acknowledging that the axial tilt of Jupiter can take the satellite out of eclipses altogether, the maximum perturbation to eclipse times is estimated as one half the duration of a central eclipse.

**Inherent variation in satellite orbital times**

The Galilean satellites are enmeshed in gravitational resonances one with another (most famously the Io, Europa and Ganymede 4:2:1 orbital resonance) and their motion is perturbed too by other bodies in the solar system, particularly the Sun and Saturn. The resulting orbital motions are complex and non-uniform.

To provide an estimate of the impact of the non-uniform orbital motion on eclipse times, Lieske’s E5 ephemeris [18] (coded as described by Meeus [19]) was used to calculate perturbation terms in longitude. (Of course, perturbations in latitude and radial distance also affect eclipse timings, but in a less direct manner, and are simply ignored here.) Running the ephemeris for the 601 year period 1600 January 01 – 2200 December 31 and recording the value of the perturbation every midnight enabled the maximum to be estimated. (The period investigated is of no special significance other than being the range spanned by DE-405, used elsewhere in the work.)

Table 5 summarises the results of the above analysis.

|  |  |  |  |
| --- | --- | --- | --- |
| **Satellite** | **Eccentricity of Orbit of Jupiter** | **Inherent Variability in Orbital Motion** | **Inclination of Galilean Orbit** |
| Io | 40 | 3.8 | 2.1 |
| Europa | 79 | 17 | 7.1 |
| Ganymede | 160 | 11 | 29 |
| Callisto | 373 | 62 | 71 |

**Table 5. Maximum effect (±minutes) of disturbing factors on times of eclipse phenomena.**

Table 5 should not be considered definitive, as the estimates are based on crude approximations and factors are considered in isolation, with no allowance for the fact that they sometimes act in concert and sometimes in opposition to one another. The maximum effect of all three factors is generally more pronounced for the outer Galileans than the inner; the exception is the inherent variability in orbital motion, for which the value for Europa exceeds that for Ganymede, due to the 1:2:4 gravitational resonance between the three innermost moons. Let T be the range of variation in times of eclipse phenomena associated with the light time between Jupiter and Earth, approximately ±10 minutes. The three factors can have maximum effects on the time of phenomena which compare in magnitude with T as follows:

* The eccentricity of Jupiter’s orbit is the dominant disturbing factor, and its maximum effect on the times of eclipse phenomena for all four Galileans, especially the outer moons, can far exceed T.
* The inherent variability in orbital motion has a maximum effect which is relatively modest for Io, is commensurate with T for Ganymede and is greater than T for Europa and Callisto (considerably so for the latter).
* The orbital inclination has a maximum effect which is modest for Io, broadly commensurate with T for Europa, and larger than T for Ganymede and Callisto (again, considerably so for the latter).

It is possible to explore the accuracy of the method further by using the E5 ephemeris to predict eclipse disappearance and reappearance mid-times then applying Method A to estimate 1/*c* from the resulting data. Figure 9 illustrates the results of the analysis for the 601-year period defined above. The points plotted represent two series of estimates of 1/*c* observed throughout each apparition of Jupiter, one based on disappearance events and the other on reappearance events. (Only disappearance events are considered after conjunction and before opposition, and only reappearance events after opposition and before conjunction; no allowance is made for the occasional visibility of both disappearance and reappearance events for eclipses of Ganymede and Callisto. Further, no allowance is made for the visibility of eclipses occurring close to conjunction or opposition being compromised due to solar glare and the proximity of the planetary limb respectively.) Note that the variability of eclipse times is such that estimates of 1/*c* can be negative, which is clearly physically unrealistic. For each estimate, the value of the coefficient of determination of the associated simple linear regression line, R2 [20] is also calculated, providing an estimate of the goodness of fit of the line (see Draper and Smith [21] for details of regression methods).

**Figure 9. Estimates of 1/*c* based on E5 model, 1600-2200. Left-right, top-bottom: Io, Europa, Ganymede, Callisto. D blue; R orange. (Note the differing scales of the y-axes.)**

The figures show complex patterns of approximately cyclic behaviour. Consider first, for Io and Europa, the series of estimates of 1/*c* based on disappearance events. Values fluctuate approximately cyclically with a period of 11.8 years, close to the sidereal period of the planet of 11.9 years. The amplitude of the fluctuations itself varies with a period of approximately three and a third centuries. Note also that the behaviour does not repeat exactly and there is some evidence of longer term trends. The series of estimates of 1/*c* based on reappearance events for the two moons display broadly similar characteristics and the two series for each satellite are in anti-phase. For Ganymede, again the two series of estimates of 1/*c* each fluctuate approximately cyclically, in anti-phase, with a period of 11.8 years. However, compared with the two inner moons, the low frequency component is much reduced and the 11.8 year periodicity dominates the overall variability. For Callisto, results are qualitatively different again and the series of values of 1/*c* illustrate eclipse seasons alternating with periods when no eclipse takes place. (It is necessary to examine the figure on a large scale for gaps between eclipse seasons to be evident.) Many outliers are visible for Callisto, all associated with entrance to or exit from eclipse seasons when there are precisely two, consecutive, eclipse events during an apparition (either both before or both after opposition). When Callisto is close to entering or leaving an eclipse season and its orbit passes near the northern or southern edge of the shadow cone, the sensitivity of the time of phenomenon to the inclination of the orbit is most acute and, when an estimate of 1/*c* is determined by only two events, it can be considerably biased. Indeed, there is a cluster of extreme values of 1/*c* every approximately 90 years on average. The range of estimates of 1/*c* is much larger for Callisto than for the other Galileans.

For all four Galileans, the majority of values of R2 are close to unity, implying that the regression lines used to generate estimates of 1/*c* fit the data well. To quantify this, choose arbitrarily the value 0.9 as a threshold value denoting “good” fit. For Io, 87% of the estimates of 1/*c* are associated with values of R2 in excess of the threshold; for Europa, fits are not so good and the value is 55%; for Ganymede and Callisto, both 89%. Low values of R2 are all associated with values of 1/*c* close to zero in absolute magnitude, with corresponding large positive or negative estimates of *c*.

Thus, it appears that the assumption that eclipse times repeat approximately periodically is generally valid within one half of an apparition of Jupiter (either preceding opposition or following it), except when orbital effects conspire to produce an unrealistically high or unrealistically low estimate of *c*, in which case, significant deviations from periodicity may occur, compromising the goodness of fit of the regression line. Disturbing factors generally operate on a longer timescale than the few months of a half-apparition before or after opposition; they may disturb the value of the synodic period of a Galilean in a consistent manner throughout a half-apparition, resulting in a regression line which fits well but delivers a sometimes highly inaccurate estimate of 1/*c*.

Method A can therefore yield inaccurate estimates of 1/*c*. Only eclipses of Io, observed over a single apparition so as to minimise perturbations of the moon’s synodic period, offer the prospect of producing a reasonable estimate if analysed according to Method A. It is possible to counteract the disturbing factors by pooling estimates over an extended period. In the case of the eccentricity of Jupiter’s orbit, observations are required over a synodic period; in the case of the other factors, the appropriate observation period remains, at present, for further study.

In fact, in Rømer’s era it was known that Jupiter’s orbit is significantly elliptical. Indeed, Cassini included a correction for ellipticity in his 1668 *Bononienses* tables. The following additional step may be added between items 2 and 3 of Method A to account for ellipticity:

2.5 Add to C the time by which the eclipse is delayed (relative to the baseline) due to the eccentricity of Jupiter’s orbit, estimated as described above.

The modified algorithm is called Method B; it represents a modern interpretation of the best approach available to Rømer and his contemporaries to estimate the speed of light, had they understood the scale of the solar system. It significantly improves the estimates and brings into phase the sequences of disappearances and reappearances; see figure 10, which corresponds to figure 9 but has the ellipticity correction in place. However, clearly the remaining uncorrected factors continue to exert a significant effect on estimates of 1/*c*, and the method is incapable of providing an accurate estimate of the speed of light without pooling data over several years or applying another technique to correct for other sources of variation of eclipse times.

**Figure 10. Estimates of 1/*c* based on E5 model, 1600-2200, with correction for ellipticity of Jupiter’s orbit. Left-right, top-bottom: Io, Europa, Ganymede, Callisto. D blue; R orange. (Note the differing scales of the y-axes.)**

Figure 11 compares theoretical and empirical estimates of the speed of light according to Method B. Theoretical results are abstracted from figure 10. Empirical estimates are from observations by members of OASI and HAG, derived from a reworking of the data used to produce table 4, applying Method B rather than Method A. The theoretical results show clearly the much greater expected variability of estimates of 1/*c*, during the period in question, for Europa and Ganymede than for Io. Observational estimates from Ganymede disappearances 2012 January 15 – 2012 February 19 and from Ganymede reappearances 2012 November 3 – 2013 March 5 clearly represent outliers. However, the other observational data is broadly in accord with theory.

**Figure 11. Comparison of estimates of 1/*c*, E5 model vs observation (with correction for ellipticity of Jupiter’s orbit).**

# Estimating *c* from observations of Rømer and Picard

There is no historical record of Rømer estimating a numerical value for the speed of light. However, his observational data, together with that of Picard, provides fertile ground for analysis by later astronomers. Kristensen and Pedersen [22] list observations by Rømer and Picard during the period 1668 October 22 to 1678 January 6. The following analysis addresses the subset of observations of Io, using the correction for location of the observing site provided by the authors.

Theoretical data, abstracted from figure 10, indicates that Method B is expected to produce estimates of 1/*c* which increase throughout the period from less than 6 min/AU at the start, to a peak of approximately 11.5 min/AU and then decline to less than 6 min/AU again at the end. The linear regression models used to generate theoretical estimates of 1/c throughout the period fit the data excellently: indeed, the minimum value of R2 for all estimates shown is 0.97.

Figure 12 illustrates the regression lines generated by an application of Method B to the observations by Rømer and Picard. In fact, the figure omits two observations which the regression analysis shows to be outliers, confirming indications from Kristensen and Pedersen that they are suspect [23]. The aqua line, corresponding to the reappearance observations on 1671 March 19, April 27 and May 4, is also troublesome, and clearly does not fit well the hypothesised linear relationship between d and e: any one of the three observations associated with the line may be regarded as an outlier, and there is no reason to prefer one over another, so all three are retained in the analysis. The remaining data (other than that associated with the aqua line just discussed) in general fits the assumed linear relationship very well; the lowest value of R2 is 0.52 and the next lowest is 0.82.

**Figure 12. Regression lines generated by applying Method B to observations by Rømer and Picard.**

Figure 13 compares the above theoretical and empirical estimates of the speed of light. As usual, estimates are provided separately for disappearances and reappearances during each apparition of the planet throughout the period on question. (Dates plotted in the figure refer to the mid-point of each set of events. Scarcity of observational data prevents calculation of empirical estimates before 1671.)

**Figure 13. Estimates of 1/*c* from eclipses of Io, 1668-78.**

Agreement between empirical data and theoretical values is clearly poor. Arguably theory and observation for disappearances at least exhibit the same broad variation but, for reappearances, even that is not the case. The discrepancy may be in part due to the difficulties described by Shea [24] and van Helden [25] of trying to derive an estimate of the speed of light from the historical observations and in part due to extrapolating the E5 ephemeris to such an early period before the observations upon which it is based.

Although individual estimates deviate significantly from theoretical predictions, weighting the former by number of observations and adding to produce a compound estimate yields a figure of 8.30 min/AU, with an associated value of *c* of 301,000 km/s. The figures represent estimates that Rømer himself could have produced, had knowledge of the scale of the solar system been available in his era; unfortunately, their satisfying proximity to modern-day accepted values are merely fortuitous.

# Conclusions

Rømer’s conclusion that the speed of light is finite, derived from observations of eclipses of the Galilean satellites of Jupiter, is one of the key results of astronomy. The amateur astronomer nowadays may readily replicate the work and, indeed, develop it further, to derive a numerical estimate of the speed of light. Surprisingly, despite the accessibility of the project and its historical resonance, there are few modern reports of astronomers tackling the task. Members of the local astronomy societies OASI and HAG provide an exception; their members, during 2012-14, undertook a project following in the footsteps of Rømer, conducted in part under the aegis of NAW 2014.

If modern knowledge of orbital dynamics of the solar system, encapsulated in the ephemerides DE-405 and E5, is applied, the observations by members of OASI and HAG may be analysed to produce a satisfyingly accurate estimate of *c*. However, an attempt to analyse the observations in a manner more faithful to that of Rømer, assuming that the Galileans can be used directly as periodic timekeepers, indicates the existence of significant perturbing factors. Analysis following the approach of Rømer, of published high-precision eclipse timings by Mallama *et al* clarifies the matter. Using crude estimates, it is possible to estimate the maximum effect of the perturbations, and analysis of eclipse predictions from the E5 ephemeris illustrates their effect on estimates of *c*. Only eclipses of Io, observed during a single apparition of Jupiter, offer the prospect of producing a reasonable estimate of *c*, if analysed in the manner of Rømer.

Analysis of the observations by Rømer and Picard of Io, 1668-78, provides estimates of *c* associated with each apparition of Jupiter. Individual estimates differ significantly from theoretical predictions; however, a compound overall estimate based on the entire data set causes a fortuitous mutual cancellation of many of discrepancies, yielding a final estimate very close to the modern accepted value. This represents an estimate that Rømer himself could have produced, had knowledge of the scale of the solar system been available in his era.

# References and notes

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